Problem Set 1

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Instructor: Fabian Eckert

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Problem 1. Gains from Trade. Consider a world with two countries. In each country there is a representative agent. There are n commodities, some of which may be intermediate goods or primary factors of production. Compare two scenarios, one in which the first k commodities are freely traded and the remaining $n-k$ are non-tradable, and another in which the first $k+m$ commodities are freely traded and the remaining $n-k-m$ are non-tradable. Use superscript 1 refer to variables in the first of these scenarios and superscript 2 refer to variables in the second.
a. Under what conditions does trade in the extra m commodities generate aggregate welfare gains for the world economy? That is, if international transfer payments are possible, when will it be possible to construct a transfer scheme so that both countries enjoy higher welfare in the latter scenario compared to the former? (Hint: construct international transfers that allow each country to purchase its 1-bundle in the 2-scenario, and check if this transfer scheme is feasible.)
b. Suppose international transfers are not possible. Under what conditions can we be sure that the home country enjoys higher aggregate welfare in the latter scenario than in the former? (Hint: use revealed-preference logic.)

Solution P1. (a.) Let the two countries be labelled A and B. Denote by p prices, x production, c consumption, $m = c - x$. Trade balance implies $p.m = 0 \Rightarrow p.c = p.x$. Also zero profits imply $Xp.x = 0$, since factors are also listed in x, e.g., if 10 labor is used up in production of outputs, then the entry for labor is -10. This is all just notation, you could have chosen a different notation.
We investigate the feasibility of a scheme of international lump-sum transfers that allows each country to purchase its 1-bundle in the 2-equilibrium. These transfers are given by:

$$p^{A2}c^{A1} = t^A$$
$$p^{B2}c^{B1} = t^B$$

But $c^{j1} = x^{j1} + m^{j1}$ for $j = A, B$, so

$$t^A + t^B = p^{A2}(x^{A1} + m^{A1}) + p^{B2}(x^{B1} + m^{B1}).$$

From profit maximization and zero-profits in equilibrium,

$$p^{A2}x^{A1} \leq p^{A2}x^{A2} = 0$$
$$p^{B2}x^{B1} \leq p^{B2}x^{B2} = 0.$$

Thus, if $p^{A2}m^{A1} + p^{B2}m^{B1} \leq 0$, then $t^A + t^B \leq 0$, and the scheme is feasible. Since $m^{A1} = -m^{B1}$, this condition becomes

$$m^{A1}(p^{A2} - p^{B2}) \leq 0$$

But $p^{A2} - p^{B2} = 0$ for the $k + m$ traded goods, and $m^{A1} = 0$ for the $n - k$ initially non-traded goods. Hence (*) is satisfied and the scheme is feasible. The extra trade always generates a world welfare gain.
(b.) For country A, if c^{A1} is affordable in the 2-equilibrium without any international transfer payments, then the country must be better off with more traded goods. We have

$$\begin{aligned} p^{A2}c^{A2} &= p^{A2}x^{A2} \\ &\geq p^{A2}x^{A1} \\ &= p^{A2}(c^{A1} - m^{A1}) \end{aligned}$$

Balanced trade implies $p^{A1}m^{A1} = 0$, so a sufficient condition for $p^{A2}c^{A2} \geq p^{A2}c^{A1}$ is

$$(p^{A2} - p^{A1})m^{A1} \leq 0. \tag{1}$$

This condition can be interpreted as: there are no aggregate terms of trade loses on the k goods traded in situation 1.

Problem 2. Non-traded Intermediate Goods. Imagine a constant-returns economy with the following production structure. There are three primary factors of production: capital, raw land, and raw labor, available in exogenously fixed supplies, K, E, and L respectively. These can be used to produce two intermediate goods: capital and raw land produce improved land, while capital and raw labor produce improved labor. Finally, improved land (B) and improved labor (H) produce food (F) and manufactures (M). The food sector is (improved)-land intensive. Let p_i be the price of factor or intermediate good i, for $i = K, E, L, B, H$. Let p be the relative price of food. Consider a country that exports manufactures and imports food in a competitive trade equilibrium. What are the distributional implications of trade in this economy? That is, consider the owners of capital, raw land and raw labor as three di§erent sets of households. Which of these households gain from trade, which lose, and under what conditions? You may use algebra, diagrams and/or logic to support your answers.

Solution P2. Let $\hat{p} < 0$ be the change in the relative price of food. First look at the stage where improved land and the labor produce final goods. This is a Heckscher-Ohlin structure. Using the standard methodology, and using the factor intensity assumption, you can derive:

$$\hat{p}_H > 0 > \hat{p} > \hat{p}_B$$

Next look at the stage where raw land and labor combine with mobile capital to produce improved land and labor. This is a specific-factor structure. Using the well-known results for the specific factors model and using the above inequality between price changes of B and H, we know

$$\hat{p}_L > \hat{p}_H > 0 > \hat{p} > \hat{p}_B > \hat{p}_E$$

So we know raw labor is a clear gainer and raw land a clear loser. As for capital, you can show that the result is ambiguous: anything can happen, depending on parameters.

Problem 3. Factor Content of Trade and Factor Prices. There are three goods and three factors. In the free trade equilibrium, country 1ís output vector is $X = (20, 20, 10)$, the world output vector (including country 1) is $X = (100, 200, 200)$ and the output price vector is $P = (1, \frac{6}{5}, 1)$. All countries have an identical constant-returns fixed-coefficient technology, with the input coefficients given by the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

where the element in row i and column j denotes the input of factor i per unit output of good j. All countries have identical homothetic preferences.
a. Calculate country 1's pattern of goods trade, and its factor content. Calculate the factor prices.
b. Suppose the price of good 1 increases. Is it the case that all three factors will either benefit or lose unambiguously, no matter what their consumption patterns happen to be? If not, which factor is real reward is affected ambiguously?

Solution P3. (a.) The country's share of world GNP is

$$s = PX/P\bar{X} = (20 + 24 + 10)/(100 + 240 + 200) = 1/10 \tag{2}$$

By identical homothetic preferences, the country's consumption vector in equilibrium must be $C' = 0.1\bar{X}' = (10, 20, 20)$. Therefore the vectors of goods trade and its factor content are

$$Z = X - C = \begin{bmatrix} 10 \\ 0 \\ -10 \end{bmatrix} \quad AZ = \begin{bmatrix} 10 \\ 0 \\ -10 \end{bmatrix}$$

When all goods are produced, factor prices satisfy $w'A = p$. Solving the three equations, which can be done by brute force, or by noting that $w' = pA^{-1}$ and

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

we find

$$w' = (1/5, 2/5, 1/5)$$

(b.) If $\hat{p}_1 > 0$, we have

$$dw_1 = (3/4)dp_1 \quad dw_2 = dw_3 = -(1/4)dp_1$$

It is clear that factors 2 and 3 lose. For 1, we need further clarification:

$$\hat{w}_1 = \frac{3}{4}\frac{p_1}{w_1}\hat{p}_1 = \frac{3}{4}\frac{1}{1/5}\hat{p}_1 = \frac{15}{4}\hat{p}_1$$